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On string models with Scherk–Schwarz supersymmetry breaking

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Abstract: We construct a general class of chiral four-dimensional string models with Scherk–Schwarz supersymmetry breaking, involving freely acting orbifolds. The basic ingredient is to combine an ordinary supersymmetry-preserving $\mathbf{Z}_N$ projection with a supersymmetry-breaking projection $\mathbf{Z}'_M$ acting freely on a subspace of the internal manifold. A crucial condition is that any generator of the full orbifold group $\mathbf{Z}_N \times \mathbf{Z}'_M$ must either preserve some supersymmetry or act freely in order to become irrelevant in some large volume limit. Tachyons are found to be absent or limited to a given region of the tree-level moduli space. We find several new models with orthogonal supersymmetries preserved at distinct fixed-points. Particular attention is devoted to an interesting $\mathbf{Z}_3 \times \mathbf{Z}'_3$ heterotic example.

Contents

1. Introduction

It is commonly accepted that weak-scale Supersymmetry (SUSY) represents a reasonable intermediate solution to the physics beyond the Standard Model (SM). However, the mechanism of SUSY breaking is still a very open issue, in particular when trying to embed the SM or its Minimal Supersymmetric version (MSSM), in a fundamental theory including gravity, such as string theory. Ultimately, we are therefore still very far from any viable example of fundamental theory for particle physics. The traditional picture for the latter is a string model with fundamental and compactification scales $M_{\rm s}$ and $M_{\rm c}$ of the order of the Planck scale, but it has now been understood that much lower $M_{\rm s}$ and/or $M_{\rm c}$ can actually be achieved [1, 2, 3], making a low SUSY-breaking scale $M_{\rm SUSY}$ more natural.

One of the most interesting mechanisms of SUSY breaking is the so-called Scherk–Schwarz (SS) mechanism [4, 5], in which SUSY is broken at $M_{\rm c}$ by twisting the boundary conditions of each field through a global R-symmetry transformation. More in general, the same idea can be used to break also gauge symmetries, by supplementing the twist with a gauge transformation [6]. This non-local breaking mechanism is very natural in the presence of compact dimensions, since the possibility of twisting boundary conditions is not forbidden by any symmetry of the theory; it can thus be considered as spontaneous, in this somewhat loose sense. Moreover, it is completely perturbative, and can therefore be efficiently investigated. These properties are quite appealing, especially from the string theory point of view, where it has been known for some time that the underlying superconformal structure forbids any continuous perturbative SUSY breaking [7]; the SS mechanism evades this theorem because SUSY is recovered only in a singular decompactification limit. As shown in [1, 8], one more interesting property arises in the string context: only massless states give a sizeable one-loop contribution to the cosmological constant. More precisely, $\Lambda \sim (n_B - n_F)\, M_{\rm c}^4 + O(e^{-M_{\rm c}^2/M_{\rm s}^2})$, where n_B and n_F denote the number of massless bosons and fermions in the model. Unfortunately, this still yields an unacceptably large value, unless $n_B = n_F$.

An important question to address for string models with SS SUSY breaking is whether $M_{\rm susy}$ can be low enough, since it is set by the compactification scale $M_{\rm c}$. In oriented models, like heterotic orbifolds, $M_{\rm c}$ is naturally tied to the string scale $M_{\rm s}$, and both must be very close to the Planck scale in order to achieve the correct value of the Newton constant and perturbative gauge couplings in $D = 4$. Gauge coupling unification is then achieved around M_s. On the contrary, this is instead generally lost by introducing a large hierarchy between M_c and M_s. Moreover, large threshold corrections to gauge couplings [9] arise in general (see [10] for studies in models with partially broken SUSY), and much effort has been devoted in the past to finding models exempt of such corrections [1, 11]. New interesting possibilities arise instead for unoriented models, where $M_{\rm c}$ and $M_{\rm s}$ are less constrained and can be independently low.

A general method to construct string models with SS-type SUSY breaking by deforming supersymmetric orbifold models [12] has been developed in the past [13]. It has been realized that basic principles of string theory, like modular invariance, pose severe restrictions on the implementation of the SS mechanism with this method, allowing in practice only discrete R-parity twists. Moreover, the range of application of this method appears to be limited to rather peculiar orbifolds [14], and unfortunately does not apply to the most interesting $\mathbf{Z}_3$ models (see [15] for a field theory analysis). Subsequently, it has been realized that similar models could be obtained as freely acting[1] orbifolds [16], opening in principle the possibility to construct a much larger class of models with SS SUSY breaking; however, little progress has been accomplished in this respect. Unoriented models can be obtained as orientifold descendants of these oriented models, and several examples have been worked out by now [17, 18].

Recently, renewed interest for the SS mechanism has emerged through a series of interesting papers considering five-dimensional orbifold field theories in which SUSY is broken at $M_{\rm c}$ to yield the SM at lower energies (see for instance [19, 20]). A particularly simple and interesting example of this kind has been obtained in [20], by compactification on an orbifold of the type $S^1/\mathbf{Z}_2 \times \mathbf{Z}_2'$, where the $\mathbf{Z}_2$ and $\mathbf{Z}_2'$ actions preserve orthogonal supersymmetries and have fixed-points separated by a translation in S^1. This construction can be reinterpreted as a SS compactification on the orbifold $S^1/\mathbf{Z}_2$, where a $\mathbf{Z}_2'$ subgroup of the R-symmetry group is used to twist the boundary conditions, or similarly with $\mathbf{Z}_2$ and $\mathbf{Z}_2'$ interchanged [20]. It represents the simplest realization of SS SUSY breaking through a freely acting orbifold, and an embedding of this simple construction into a realistic string model is a very important challenge for the future.

The aim of this paper is to investigate in some generality the possibility of constructing orbifold string models with SUSY broken à la SS, in which orthogonal supersymmetries are preserved at distinct fixed-points separated by a translation in the compact space. The main idea is to combine a standard SUSY-preserving action G with a SUSY-breaking action G' acting freely in a subspace of the internal compactification torus, in such a way that the generators of the full $G \times G'$ action consist of SUSY-preserving elements with disjoint sets of fixed-points, and freely acting SUSY-breaking elements mapping the fixed-points of each set into each other. Possible tachyons can arise only in twisted sectors of the freely acting elements, and are therefore massive over most of the compactification moduli space. We find that the allowed geometries are basically either of the known $\mathbf{Z}_2 \times \mathbf{Z}_2'$ type, possibly with an additional $\mathbf{Z}_K$ projection, or of a new $\mathbf{Z}_3 \times \mathbf{Z}_3'$ type, and we construct a general class of examples of this kind of $\mathbf{Z}_N \times \mathbf{Z}_N'$ models.

[1] By freely acting, we always mean an action that is free at least on a submanifold of the compactification manifold.

As an interesting application, we present a novel class of four-dimensional $\mathbf{Z}_3 \times \mathbf{Z}'_3$ heterotic models with SS SUSY breaking. For generic embedding of the orbifold action in the gauge bundle, one finds a chiral spectrum with a non-vanishing $n_B - n_F$ which can be either positive or negative, but not zero. Also the number n_T of would-be tachyons is in general non-zero, but there are a few examples with $n_T = 0$, for which tachyons are therefore completely absent.

The paper is organized as follows. In section 2, we describe the main features of freely acting orbifolds that are relevant to our construction. In section 3, the partition functions of such models are derived and in section 4 their stability is briefly analysed. In section 5 we present an explicit $\mathbf{Z}_N \times \mathbf{Z}'_N$ construction, discuss their realization for $N = 2, 3$, and in section 6 we consider in more detail a $\mathbf{Z}_3 \times \mathbf{Z}'_3$ example. In the last section we report our conclusions. Some modular properties of orbifold partition functions are collected in an appendix.

2. Freely acting orbifolds and SUSY breaking

The type of orbifold models we are looking for can be described in very simple terms. Consider an orbifold group G generated by a set of elements $\{g_i, g'_j\}$ such that each g_i acts non-freely, with fixed-points $P^k_{g_i}$, and preserves some SUSY, whereas the g'_j act freely, and do not preserve any SUSY. Clearly, such a construction is highly constrained from the requirements of a finite group structure and modular invariance. Moreover, the freely acting generators g'_j should map a fixed-point $P^k_{g_i}$ of any of the non-freely acting element g_i into another fixed-point $P^{k'}_{g_i}$ of the same element g_i; this condition ensures, in particular, that the orbifold is abelian.

The crucial property characterizing SUSY breaking in such a model is the fact that the associated elements act freely. More precisely, they must act as a simple translation by a finite fraction of lattice vector in at least one of the 3 internal tori. This implies indeed that such elements trivialize in a suitable decompactification limit, in which SUSY is therefore restored. This is a clear implementation of the SS SUSY breaking mechanism in string theory and, interestingly enough, the same mechanism can be applied also to gauge symmetries by embedding non-trivially the g'_j's in the gauge bundle. Intuitively, it is obvious that thanks to the translation that they contain, the elements g'_j result effectively in the implementation of a twist around a given cycle of the internal space. Would the g'_j's act non-freely, then SUSY would be broken at the string scale rather than the compactification scale.

In the following, we construct explicit examples of the above type by considering product groups $G = \mathbf{Z}_M \times \mathbf{Z}'_N$, in which the $\mathbf{Z}_M$ factor is generated by a non-freely acting SUSY-preserving element g, and the factor $\mathbf{Z}'_N$ by a freely acting SUSY-breaking element g'. In order to obtain the required structure, one must then analyse the action of each generator of the full G. To this aim, it is convenient to recall at this stage some basic facts about supersymmetries in four-dimensional orbifold compactifications. The basic Majorana–Weyl supercharge Q in $D = 10$ fills the $\mathbf{16}$ of $SO(9,1)$. This decomposes in $D = 4$ into four Majorana supercharges $Q_n = Q_{nL} + Q_{nR}$, transforming each as a $\mathbf{2} \oplus \bar{\mathbf{2}}$ under $SO(3,1)$ and together as a $\mathbf{4}$ of the maximal $SO(6)$ R-symmetry group. For each $n = 1,2,3,4$, Q_{nL} and Q_{nR} have $SO(6)$ weights w_n and $-w_n$ respectively, where:

$$w_1 = (\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2})\ ,\quad w_2 = (\tfrac{1}{2},-\tfrac{1}{2},-\tfrac{1}{2})\ ,\quad w_3 = (-\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2})\ ,\quad w_4 = (-\tfrac{1}{2},-\tfrac{1}{2},\tfrac{1}{2})\ .$$

A generic orbifold element g acts as the combination of a rotation of angle $2\pi v_i$ and some unspecified shift, in each of the 3 internal T^2_i. Under this action, the 4 possible supercharges transform as:

$$\begin{aligned} Q_{nL} &\rightarrow e^{2\pi i v\cdot w_n} Q_{nL}\ , \\ Q_{nR} &\rightarrow e^{-2\pi i v\cdot w_n} Q_{nR}\ . \end{aligned}$$

Therefore, the supercharge Q_n is left invariant by g if $v \cdot w_n$ is an integer, independently of the shift.

3. Partition functions

In order to construct explicit examples of the models described in the previous section, the basic two-dimensional blocks of the partition function for a generic twist involving both a rotation and a translation are needed. These are used to derive the full heterotic and Type II partition functions of such orbifolds and deduce the constraints arising from the requirement of modular invariance. Type I open descendants could be constructed in the usual way, with the additional constraint of tadpole cancellation, but we shall not consider such constructions here. Most of the results reported below are standard [21, 22], but we have re-analysed them without any assumption about SUSY, in order to avoid any possible confusion.

Consider first a $\mathbf{Z}_N$ group generated by the element $\alpha = \alpha_{\rm geom}\alpha_{\rm gauge}$, where $\alpha_{\rm geom}$ defines the geometric action on the internal compactification torus and $\alpha_{\rm gauge}$ is its embedding in the gauge bundle. Take the geometric part to be

$$\alpha_{\rm geom} = \exp 2\pi i \sum_{i=1}^{3} \left(v_i J^i + R_i \delta_i P^i \right) , \tag{3.1}$$

with J^i and P^i being the generators of rotations and diagonal translations in each internal two-torus T^2_i with basic radii R_i. The gauge part is of course trivial for Type IIB models, whereas for the $E_8 \times E_8$ heterotic string, it has the general form

$$\alpha_{\rm gauge} = \exp 2\pi i \sum_{p=1}^{8} \left(v'_p J'^p + v''_q J''^q \right) , \tag{3.2}$$

with J'^p and J''^q being the Cartan currents of the two E_8 factors.

In order to have $\alpha^N = 1$, one must take $v_i = r_i/N$, $v'_p = r'_p/N$ and $v''_q = r''_q/N$ with integer r_i, r'_p and r''_q, and due to spinor representations, impose the constraints

$$N \Big(\sum_{i=1}^{3} v_i, \sum_{p=1}^{8} v'_p, \sum_{q=1}^{8} v''_q \Big) = 0 \bmod 2 \,. \tag{3.3}$$

The partition function for one complex field with twists $(g,h) = (kv, lv)$, shifts $(\hat g, \hat h) = (k\delta, l\delta)$, and spin structure (a,b) $(k,l = 0,1,...,N-1;\ a,b = 0, \frac{1}{2})$ is easily computed. For a complete (left + right) boson, one finds:

$$Z_B \begin{bmatrix} h \,\big|\, \hat h \\ g \,\big|\, \hat g \end{bmatrix} (\tau) = \begin{cases} |\eta(\tau)|^{-4} \Lambda \begin{bmatrix} \hat h \\ \hat g \end{bmatrix} (\tau) & , \ \text{if } (g,h) = (0,0) \\ \left| \eta(\tau)\, \theta^{-1} \begin{bmatrix} \frac{1}{2} + h \\ \frac{1}{2} + g \end{bmatrix} (\tau) \right|^2 & , \ \text{if } (g,h) \neq (0,0) \end{cases} .$$

The lattice contribution is given by (see [16]):

$$\begin{aligned} \Lambda \begin{bmatrix} \hat h \\ \hat g \end{bmatrix} (\tau) &= \frac{\sqrt{G}}{\alpha' \, {\rm Im}\, \tau} \sum_{\vec m, \vec n} e^{-\frac{\pi}{\alpha' {\rm Im}\, \tau} [(m+\hat g) + (n + \hat h)\tau]_i (G+B)_{ij} [(m+\hat g)+(n+\hat h)\bar\tau]_j} \\ &= \sum_{\vec m, \vec n} e^{2\pi i \, \hat g \cdot m} \, q^{\frac{1}{2}|P_L[\hat h]|^2} \, \bar q^{\frac{1}{2}|P_R[\hat h]|^2} \, , \end{aligned} \tag{3.4}$$

where $\sqrt{G} = \sqrt{{\rm det} G_{ij}}$ is related to the volume V of the target-space torus T^2 by $V = (2\pi)^2 \sqrt{G}$ and the lattice momenta are given by

$$\begin{aligned} P_L[\hat h] &= \frac{1}{\sqrt{2\, {\rm Im}\, T \, {\rm Im}\, U}} \Big[- m_1 \, U + m_2 + T \left((n_1 + \hat h) + (n_2 + \hat h) \, U \right) \Big] \, , \\ P_R[\hat h] &= \frac{1}{\sqrt{2\, {\rm Im}\, T \, {\rm Im}\, U}} \Big[- m_1 \, U + m_2 + \bar T \left((n_1 + \hat h) + (n_2 + \hat h) \, U \right) \Big] \, , \end{aligned} \tag{3.5}$$

in terms of the standard dimensionless moduli T and U parametrizing the metric G and antisymmetric field B as:

$$G_{ij} = \alpha' \frac{\mathrm{Im}\, T}{\mathrm{Im}\, U} \begin{pmatrix} 1 & \mathrm{Re}\, U \\ \mathrm{Re}\, U & |U|^2 \end{pmatrix} \; , \quad B_{ij} = \alpha' \begin{pmatrix} 0 & \mathrm{Re}\, T \\ -\mathrm{Re}\, T & 0 \end{pmatrix} \; . \tag{3.6}$$

Notice that (3.4) reduces to $V/(4\pi^2\alpha' \,\mathrm{Im}\, \tau)$ for a non-compact boson.

For a fermion, one finds instead

$$\begin{aligned} Z_F \begin{bmatrix} a & h \\ b & g \end{bmatrix} (\tau) &= \eta^{-1}(\tau)\, e^{-2\pi i b h}\, \theta \begin{bmatrix} a+h \\ b+g \end{bmatrix} (\tau) \\ &= \eta^{-1}(\tau) \sum_{p_a = n+a} q^{\frac{1}{2}(p_a+h)^2} e^{2\pi i (p_a+h) g} e^{2\pi i p_a b} \, . \end{aligned} \tag{3.7}$$

Notice in particular the crucial phase in the first row of (3.7)[2], ensuring that the GSO projection amounts to the standard constraints on the bosonization momentum p independently of h, as evident from the second row of (3.7).

The modular properties of these basic partition functions, as well as the constraints they impose, are derived in the appendix; we report here only the main final results. Denoting the generic twist with $G = (g_i, \hat{g}_i, g_i', g_i'')$ and its conjugate with $\bar{G} = (Nv_i - g_i, 1 - \hat{g}_i, Nv_i' - g_i', Nv_i'' - g_i'')$, the total partition function[3] is given by

$$Z = \sum_{G,H} C \begin{bmatrix} H \\ G \end{bmatrix} N \begin{bmatrix} H \\ G \end{bmatrix} Z \begin{bmatrix} H \\ G \end{bmatrix} , \tag{3.8}$$

where $Z\left[{H \atop G}\right]$, $N\left[{H \atop G}\right]$ and $C\left[{H \atop G}\right]$ denote respectively the partition function, the number of fixed-points and an arbitrary overall phase in each sector $\left[{H \atop G}\right]$.

For a generic Type IIB model, one finds

$$Z \begin{bmatrix} H \\ G \end{bmatrix} = Z_B \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \prod_{i=1}^{3} Z_B \begin{bmatrix} h_i & \hat{h}_i \\ g_i & \hat{g}_i \end{bmatrix} \left| \sum_{a,b=0,\frac{1}{2}} \eta_{ab}\, Z_F \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} \prod_{i=1}^{3} Z_F \begin{bmatrix} a & h_i \\ b & g_i \end{bmatrix} \right|^2 , \tag{3.9}$$

$$C \begin{bmatrix} H \\ G \end{bmatrix} = 1 \, , \tag{3.10}$$

where $\eta_{ab} = (-1)^{2a+2b+4ab}$ are the usual GSO-projection signs. The partition function is then modular-invariant without any further restriction.

[2]This phase is very often missing in the literature, probably because it is irrelevant for supersymmetric constructions. It has previously been noticed in [22], but is not evident in [21].

[3]To be precise, we consider the light-cone partition function; the contribution of longitudinal degrees of freedom provides only the correct invariant measure $d^2\tau/\mathrm{Im}\,\tau^2$ in the world-sheet moduli space.

For a generic heterotic $E_8 \times E_8$ model one gets:

$$Z\begin{bmatrix} H \\ G \end{bmatrix} = Z_B\begin{bmatrix} 0 \\ 0 \end{bmatrix}\!\!\begin{bmatrix} 0 \\ 0 \end{bmatrix} \prod_{i=1}^{3} Z_B\begin{bmatrix} h_i & \hat{h}_i \\ g_i & \hat{g}_i \end{bmatrix}\Big(\frac{1}{2}\sum_{a,b=0,\frac{1}{2}} \eta_{ab}\, Z_F\begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} \prod_{i=1}^{3} Z_F\begin{bmatrix} a & h_i \\ b & g_i \end{bmatrix}\Big)$$

$$\times\Big(\frac{1}{2}\sum_{c,d=0,\frac{1}{2}} \prod_{p=1}^{8} \bar{Z}_F\begin{bmatrix} c & h'_p \\ d & g'_p \end{bmatrix} \times \frac{1}{2}\sum_{e,f=0,\frac{1}{2}} \prod_{q=1}^{8} \bar{Z}_F\begin{bmatrix} e & h''_q \\ f & g''_q \end{bmatrix}\Big), \tag{3.11}$$

$$C\begin{bmatrix} H \\ G \end{bmatrix} = e^{-i\pi(g\cdot h - g'\cdot h' - g''\cdot h'')}\,. \tag{3.12}$$

This partition function is modular-invariant provided that the embedding satisfy the condition [21, 22] (see also [23]):

$$N(v^2 - v'^2 - v''^2) = 0 \bmod 2\,. \tag{3.13}$$

It is straightforward to extend this analysis to more general orbifold groups G with more than one generator. The results (3.9)-(3.12) hold true, but the condition (3.13) must be extended to all the independent generators of G.

Recall finally that modular invariance of the partition function, together with the condition of tadpole cancellation for unoriented descendants, guarantees the full consistency of this kind of string models, and implies in particular the absence of divergences or anomalies. Actually, as shown in [24, 25], even the complete mechanism of anomaly cancellation is encoded in the background dependence of the partition function itself in a very natural way.

4. Stability

An important issue for the kind of non-supersymmetric models we aim to construct is their stability. At tree level, one must make sure that no tachyonic modes appear.

To check the presence of tachyons, one must compute the zero-point energy. The contribution of each left or right complex field can be easily read off from the behaviour of the corresponding partition function in the limit $\mathrm{Im}\,\tau \to \infty$. One finds:

$$E_B^0[h] = \frac{1}{24} - \frac{1}{2}\Big(\frac{1}{2} - \theta[\tfrac{1}{2}|h]\Big)^2, \tag{4.1}$$

$$E_F^0[a|h] = -\frac{1}{24} + \frac{1}{2}\Big(a - \theta[a|h]\Big)^2, \tag{4.2}$$

where

$$\theta[a|h] = |h| - \mathrm{int}(|h| + \tfrac{1}{2} - a)\,. \tag{4.3}$$

In the following, it will prove convenient to use the bosonized description for all the fermions. Correspondingly, one must count a zero-point energy of only $E_F^0[0|0] = -1/24$ for each of them, since in this description the orbifold action is a lattice shift and no longer a twist. It is then useful to define the quantity

$$C[h] = \frac{1}{2}\sum_{i=1}^{3}\theta[\tfrac{1}{2}|h_i]\Big(1-\theta[\tfrac{1}{2}|h_i]\Big)\,. \tag{4.4}$$

For Type IIB models, the mass formula is:

$$\begin{aligned} L_0[a|h] &= \frac{1}{2}|P_L[\hat{h}]|^2 + N_L[h] + \frac{1}{2}(p_a+h)^2 + \Big(-\frac{1}{2}+C[h]\Big)\,, \\ \bar{L}_0[c|h] &= \frac{1}{2}|P_R[\hat{h}]|^2 + N_R[h] + \frac{1}{2}(p_c+h)^2 + \Big(-\frac{1}{2}+C[h]\Big)\,. \end{aligned}$$

For heterotic models, one finds similarly:

$$\begin{aligned} L_0[a|h] &= \frac{1}{2}|P_L[\hat{h}]|^2 + N_L[h] + \frac{1}{2}(p_a+h)^2 + \Big(-\frac{1}{2}+C[h]\Big)\,, \\ \bar{L}_0[c|e|h] &= \frac{1}{2}|P_R[\hat{h}]|^2 + N_R[h] + \frac{1}{2}(p_c^\prime+h^\prime)^2 + \frac{1}{2}(p_e^{\prime\prime}+h^{\prime\prime})^2 + \Big(-1+C[h]\Big)\,. \end{aligned}$$

The sectors a, c, e are now associated to the different classes of lattice vectors.

For each state, the Hamiltonian $H[h] = L_0[h] + \bar{L}_0[h]$ gives the mass squared as $m^2[h] = \frac{2}{\alpha'}H[h]$, whereas the level mismatch $\Delta[h] = L_0[h] - \bar{L}_0[h]$ is related by modular invariance to the phase picked under the orbifold transformation, which reads $\phi(g) = e^{2\pi i \Delta[g]}$ for an orbifold transformation g[4]. The level-matching condition $L_0[h] = \bar{L}_0[h]$ therefore implies invariance under the orbifold action in twisted sectors.

Tachyons can occur only in twisted sectors associated to the SUSY-breaking elements. The mass of a generic state in these sectors is of the form $m^2 = m_0^2 + \frac{1}{\alpha'}(|P_L|^2 + |P_R|^2)$, with a moduli-independent contribution m_0^2, which can be either positive or negative, but a positive-definite moduli-dependent contribution from the internal momentum. It is then quite clear that possible tachyons can always be made massive by selecting a suitable part of moduli space. More precisely, the minimal value of $|P_L|^2 + |P_R|^2$ is generically obtained for the zero mode $m = n = 0$; one then finds:

$$m^2 > m_0^2 + \frac{\hat{h}^2}{\alpha'}\frac{|T(1+U)|^2}{\mathrm{Im\,T\,Im\,U}}\,. \tag{4.5}$$

Focusing for concreteness on a fixed complex structure U, the condition for the absence of tachyons turns into a restriction on the Kähler modulus T. One finds:

$$(\mathrm{Re}\,T)^2 + (\mathrm{Im}\,T - T_0)^2 > T_0^2\,, \tag{4.6}$$

[4]Notice for instance that the contribution to $\Delta[g]$ from KK and winding modes is given by $\frac{1}{2}(|P_L[\hat{g}]|^2 - |P_R[\hat{g}]|^2) = m\cdot(n+\hat{g})$ and leads to the phase $e^{2\pi i m\cdot\hat{g}}$, in agreement with (3.4).

with

$$T_0 = \frac{\alpha'(-m_0^2)}{2\,\hat{h}^2}\frac{\mathrm{Im\,U}}{|1+U|^2}\,. \tag{4.7}$$

This condition excludes a circle close to the origin in the moduli space of $T = (B + i\,R^2)/\alpha'$, as depicted in fig. 1. Notice in particular that only $R \geq \sqrt{T_0\,\alpha'}$ is allowed for $B = 0$, but all the $R > 0$ are allowed as soon as $B \geq T_0\,\alpha'$.

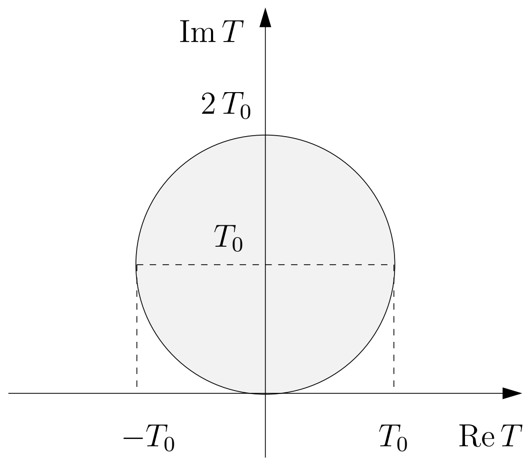

Figure 1: Tachyons can arise only in the shaded region of the T moduli space.

It is clear from the above discussion that the stability of this kind of model is triggered by the effective potential of both the radion field and its pseudoscalar partner, entering together in a would-be chiral multiplet T of the low-energy effective action. This potential is completely flat at the tree level, but since SUSY is broken, non-trivial quantum corrections are expected to occur, and the VEV of T is dynamically fixed.

5. Explicit constructions

The general construction described so far actually admits relatively few concrete realizations. We restrict ourselves to abelian orbifolds involving rotations and translations in the lattice of the internal T^6. The basic group structure is of the form $\mathbf{Z}_M \times \mathbf{Z}'_N$, where $\mathbf{Z}_M$ is generated by a standard SUSY-preserving non-freely acting rotation α, and $\mathbf{Z}'_N$ by the combination β of a SUSY-breaking rotation and a translation. We focus in the following on the case $M = N$.

The partition function of such $\mathbf{Z}_N(\alpha) \times \mathbf{Z}'_N(\beta)$ models has the form:

$$Z = \frac{1}{N^2} \sum_{k,l=0}^{N-1} \sum_{p,q=0}^{N-1} N \begin{bmatrix} \alpha^l \beta^q \\ \alpha^k \beta^p \end{bmatrix} Z \begin{bmatrix} \alpha^l \beta^q \\ \alpha^k \beta^p \end{bmatrix} . \tag{5.1}$$

Interestingly, this partition function can be decomposed into $\mathbf{Z}_N$ blocks. To see this, let us define $\alpha_i = \alpha\beta^{i-1}$, $i = 1, \cdots, N$, so that the total orbifold group can be rewritten as $G = \{1, \alpha_i, \cdots, \alpha_i^{N-1}, \beta, \cdots, \beta^{N-1}\}$. The new elements α_i are combinations of SUSY-preserving rotations and translations, with fixed-points $P^a_{\alpha_i}$ differing from one another by the translation contained in β (see figs. 2 and 3). In this new parametrization, the partition function simplifies substantially: only those sectors of the form $\left[\begin{smallmatrix} \alpha_i^q \\ \alpha_i^p \end{smallmatrix}\right]$ or $\left[\begin{smallmatrix} \beta^q \\ \beta^p \end{smallmatrix}\right]$ give a non-vanishing contribution to the partition function. In this sense, one can therefore write: $\mathbf{Z}_N(\alpha) \times \mathbf{Z}'_N(\beta) = \mathbf{Z}_N(\alpha_1) + \cdots + \mathbf{Z}_N(\alpha_{N-1}) + \mathbf{Z}_N(\beta)$, and the partition function can be rewritten as:

$$Z = Z^U_{\mathbf{Z}_N(\alpha_i)\times\mathbf{Z}_N(\beta)} + \sum_{i=1}^{N} \frac{N_{\mathbf{Z}_N(\alpha_i)}}{N} Z^T_{\mathbf{Z}_N(\alpha_i)} + \frac{N_{\mathbf{Z}_N(\beta)}}{N} Z^T_{\mathbf{Z}_N(\beta)} \,, \tag{5.2}$$

where

$$Z^U_{\mathbf{Z}_N(\alpha_i)\times\mathbf{Z}_N(\beta)} = \frac{1}{N^2}\left[Z\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \sum_{b=1}^{N-1} \left(\sum_{i=1}^{N} Z\begin{bmatrix} 0 \\ \alpha_i^b \end{bmatrix} + Z\begin{bmatrix} 0 \\ \beta^b \end{bmatrix} \right)\right] , \tag{5.3}$$

$$Z^T_{\mathbf{Z}_N(\alpha_i)} = \frac{1}{N} \sum_{a=1}^{N-1} \sum_{b=0}^{N-1} Z\begin{bmatrix} \alpha_i^a \\ \alpha_i^b \end{bmatrix} , \tag{5.4}$$

$$Z^T_{\mathbf{Z}_N(\beta)} = \frac{1}{N} \sum_{a=1}^{N-1} \sum_{b=0}^{N-1} Z\begin{bmatrix} \beta^a \\ \beta^b \end{bmatrix} . \tag{5.5}$$

The untwisted sector can be computed by projecting the usual untwisted sector of any of the supersymmetric $\mathbf{Z}_N(\alpha_i)$ orbifolds with the SUSY-breaking action $\mathbf{Z}_N(\beta)$. The choice of α_i is irrelevant, since $\mathbf{Z}_N(\alpha_i) \times \mathbf{Z}_N(\beta) = G$ for any α_i. The massless states of the $\mathbf{Z}_N(\alpha_i)$ orbifold that are not invariant under $\mathbf{Z}_N(\beta)$ will survive only as KK or winding modes, and get a mass of order M_c. There are then the twisted sectors of the supersymmetric $\mathbf{Z}_N(\alpha_i)$ orbifolds and those of the non-superymmetric $\mathbf{Z}_N(\beta)$ orbifold, all with a degeneracy given by the number of fixed-points divided by a factor N. This additional factor has a clear geometric interpretation. For α_i twisted sectors, it reflects the fact that not all the fixed-points are independent; they fall into groups of N filling orbits of β. For β twisted sectors, all the fixed-hyperplanes are independent, but they fill the T^2 where the shift acts, and there is therefore an additional factor of $1/N$ from the volume.

In these models, SUSY is broken at a scale set by the volume of the T^2 where β acts as a shift: $M_{\rm SUSY} = M_c$. This is due to the shift entering the definition of β. Indeed, this has two crucial consequences. The first is that the SUSY-breaking element β trivializes for large R. The second is that the N elements α_i, preserving different fractions of the maximal SUSY, have fixed-points which differ by a fraction of lattice vectors, and therefore move far apart for large R. Potential tachyons can appear only in the non-supersymmetric $\mathbf{Z}_N(\beta)$ twisted sectors. As explained in section 4, all the states in these sectors have a moduli-dependent positive contribution to their mass squared, and tachyons can thus always be avoided.

Out of these basic $\mathbf{Z}_N \times \mathbf{Z}'_N$ models, one can then in general construct more complicated models by further projecting with an additional $\mathbf{Z}_K$ action generated by a SUSY-preserving rotation γ, which is orthogonal to the translation in β. We report in the following the examples that we have been able to construct.

5.1 $\mathbf{Z}_2 \times \mathbf{Z}'_2$ models

Consider the orbifold group $G = \mathbf{Z}_2 \times \mathbf{Z}'_2$, where the two factors are generated by the elements

$$\alpha : v_\alpha = (\tfrac{1}{2}, \tfrac{1}{2}, 0) \;, \quad \delta_\alpha = (0,0,0) \,; \tag{5.6}$$

$$\beta : v_\beta = (0,1,0) \;, \quad \delta_\beta = (\tfrac{1}{2}, 0, 0) \,. \tag{5.7}$$

This kind of models have already been considered in [16, 17]. Defining $\alpha_i = \alpha\beta^{i-1}$, the orbifold group can be rewritten as $G = \{1, \alpha_i, \beta\}$, where

$$\begin{aligned} &\alpha_1 : \text{preserves } Q_2 \text{ and } Q_3 \,; \\ &\alpha_2 : \text{preserves } Q_1 \text{ and } Q_4 \,; \\ &\beta : \;\; \text{does not preserve any } Q_n \,. \end{aligned} \tag{5.8}$$

This model can be lifted to $D = 6$, where it represents the unique possibility of a model with $N = 1 \to N = 0$ SUSY breaking. From the $D = 4$ point of view, however, it has $N = 2 \to N = 0$ SUSY breaking and is therefore non-chiral. More interesting $D = 4$ models with orbifold group $G = \mathbf{Z}_2 \times \mathbf{Z}_K \times \mathbf{Z}'_2$ can be obtained by a further $\mathbf{Z}_K$ orbifold projection acting in the last two T^2's, which does not influence the freely acting SUSY-breaking element. One can choose the generator γ of this action to have $v_\gamma = (0, \frac{1}{K}, \frac{1}{K})$ and $\delta_\gamma = (0,0,0)$. It is then straightforward to show that all the elements in G either preserve some SUSY or act freely in the first T^2. More precisely, for $k = 1, ..., K-1$, one finds that, in addition to the conditions

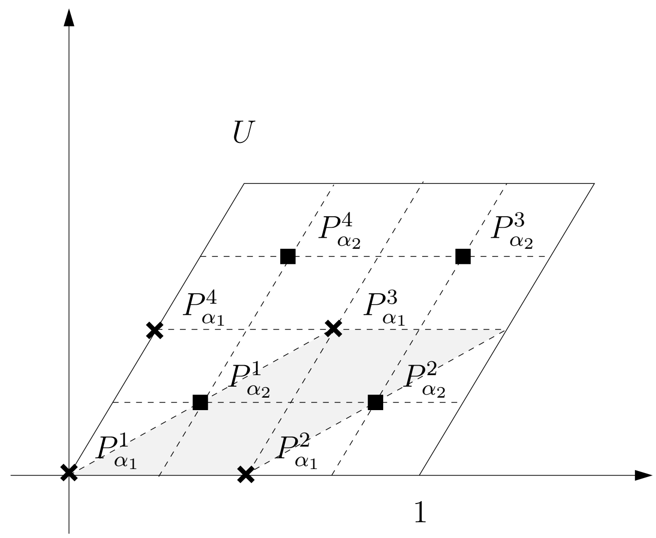

Figure 2: The α_i fixed-points $P^a_{\alpha_i}$ in T^2_1 for the $\mathbf{Z}_2 \times \mathbf{Z}'_2$ model. The SUSY-breaking element β acts as a shift in this plane, and relates different fixed-points of the same element α_i, $\beta : P^a_{\alpha_i} \to P^{a+2}_{\alpha_i}$. One can take $P^{1,2}_{\alpha_i}$ as independent fixed-points. Correspondingly, the fundamental cell of the orbifold theory can be chosen to be the shaded area, since this is mapped to the fundamental cell of the whole torus through α_i and β transformations.

(5.8), γ^n preserves Q_3 and Q_4, $\alpha\gamma^k$ preserves at least Q_3, $\alpha\gamma^k\beta$ preserves at least Q_4, whereas $\gamma^n\beta$ do not preserve any Q_n in general but act as translations in the first T^2. The resulting models have therefore all $N = 1 \to N = 0$ SUSY breaking. The cases $K = 2, 3$ have already been discussed in [18].

5.2 $\mathbf{Z}_3 \times \mathbf{Z}'_3$ models

Consider now the orbifold group $G = \mathbf{Z}_3 \times \mathbf{Z}'_3$, where the two factors are generated by the elements:

$$\alpha : v_\alpha = (\tfrac{1}{3}, \tfrac{1}{3}, 0)\ , \quad \delta_\alpha = (0, 0, 0)\,; \tag{5.9}$$

$$\beta : v_\beta = (0, 0, \tfrac{2}{3})\ , \quad \delta_\beta = (\tfrac{1}{3}, 0, 0)\,. \tag{5.10}$$

Different choices for the SUSY-preserving element α lead to equivalent models, and no other options are possible for the SUSY-breaking element β, so that this construction is essentially unique. For instance, an equivalent model would have been obtained by considering the usual $N = 1$ supersymmetric $\mathbf{Z}_3$ twist for v_α. Defining $\alpha_i = \alpha\beta^{i-1}$, the total orbifold group can be rewritten as $G = \{1, \alpha_i, \alpha_i^2, \beta, \beta^2\}$, where:

$$\begin{aligned}
&\alpha_1 : \text{preserves } Q_2 \text{ and } Q_3\,; \\
&\alpha_2 : \text{preserves } Q_4\,; \\
&\alpha_3 : \text{preserves } Q_1\,; \\
&\beta : \ \ \text{does not preserve any } Q_n\,.
\end{aligned} \tag{5.11}$$

Notice that this class of models does not have $N = 2$ twisted sectors along the SUSY-breaking directions, implying that most likely no threshold corrections will depend on the corresponding moduli.

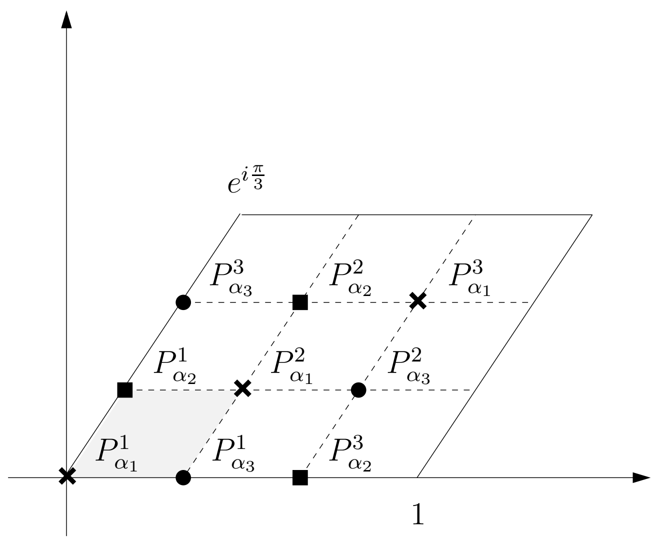

Figure 3: The α_i fixed-points $P^a_{\alpha_i}$ in T^2_1 for the $\mathbf{Z}_3 \times \mathbf{Z}'_3$ model. Again, β acts as a shift in this plane, and relates different fixed-points of the same α_i, $\beta : P^a_{\alpha_i} \to P^{a+1}_{\alpha_i}$. One can take $P^1_{\alpha_i}$ as independent fixed-points and the shaded area as the fundamental cell of the orbifold theory.

6. The $\mathbf{Z}_3 \times \mathbf{Z}'_3$ heterotic model

Consider the $\mathbf{Z}_3 \times \mathbf{Z}'_3$ construction introduced in the previous section applied to heterotic strings. The condition (3.13), evaluated for all the independent generators, implies that the embeddings should satisfy:

$$v'^2_\alpha + v''^2_\alpha = \frac{2}{9} \bmod \frac{2}{3}\,, \quad v'^2_\beta + v''^2_\beta = \frac{4}{9} \bmod \frac{2}{3}\,, \quad v'_\alpha v'_\beta + v''_\alpha v''_\beta = 0 \bmod \frac{2}{3}\,. \tag{6.1}$$

We will analyse in detail the case of standard embedding of both actions into the gauge bundle as an example, and then discuss qualitative features of more general embeddings.

6.1 Standard embedding

Consider first the untwisted sector. This is most easily derived starting from the $N=2$ $\mathbf{Z}_3(\alpha)$ model, and further projecting the spectrum by $\mathbf{Z}_3(\beta)$. Massless left-moving states are associated with lattice 4-vectors p with $p^2=1$ and $\sum_m p_m =$ odd, filling the $\mathbf{8_V}$ and $\mathbf{8_S}$ of $SO(8)$. These states pick up a phase $\phi_\alpha(p)=e^{2\pi i p\cdot v_\alpha}$ under α transformations, and a phase $\phi_\beta(p)=e^{2\pi i p\cdot v_\beta}$ under β transformations. Denoting with $\alpha=e^{\frac{2\pi}{3}i}$ and $\beta=e^{\frac{2\pi}{3}i}$ the basic phases under these two transformations, one finds the following decomposition:

$$
\begin{aligned}
\mathbf{8_V} &\to \Big[\mathbf{2_V}\Big] \oplus \Big[2\cdot\mathbf{1}\Big](\alpha+\alpha^{-1}) \oplus \Big[\mathbf{1}\Big](\beta+\beta^{-1})\,, \\
\mathbf{8_S} &\to \Big[2\cdot\mathbf{1}\Big](\beta+\beta^{-1}) \oplus \Big[\mathbf{1}\Big](\alpha+\alpha^{-1})(\beta+\beta^{-1})\,.
\end{aligned}
\tag{6.2}
$$

There are then several relevant types of right-moving states. Neutral states arise from right-moving states with $p=p'=p''=0$ and $N_R=1$, and fill an $\mathbf{8_V}$ of $SO(8)$. Under the orbifold action, they decompose as their left-mover counterparts. Charged states under each E_8 factor are instead associated to right-moving lattice 8-vectors p' or p'' with $p'^2, p''^2=0,2$ and $\sum_m p'_m =$ even, corresponding to $N_R=1,0$. These fill a $\mathbf{120}$ and a $\mathbf{128}$ of $SO(16)$, forming in total the $\mathbf{248}$ of E_8. The hidden sector is unaffected by the orbifold projection. In the visible sector, the $\mathbf{Z}_3(\alpha)$ projection breaks E_8 to $E_7\times U(1)$, whereas the $\mathbf{Z}_3(\beta)$ projection further breaks this to $SO(10)\times SU(2)\times U(1)^2$ and makes all the gauginos massive; charged states pick up a phase $\phi_\alpha(p')=e^{-2\pi i p'\cdot v_\alpha}$ under α transformations, and a phase $\phi_\beta(p)=e^{-2\pi i p'\cdot v_\beta}$ under β transformations, and decompose as follows[5]:

$$
\begin{aligned}
\mathbf{120} &\to \Big[(\mathbf{45},\mathbf{1})\oplus(\mathbf{1},\mathbf{3})\oplus 2\,(\mathbf{1},\mathbf{1})\Big] \oplus \Big[(\mathbf{10},\mathbf{2})\oplus(\mathbf{1},\mathbf{1})\Big](\alpha+\alpha^{-1}) \\
&\quad \oplus \Big[(\mathbf{10},\mathbf{1})\Big](\beta+\beta^{-1}) \oplus \Big[(\mathbf{1},\mathbf{2})\Big](\alpha+\alpha^{-1})(\beta+\beta^{-1})\,, \\
\mathbf{128} &\to \Big[(\mathbf{16},\mathbf{2})\Big](\beta+\beta^{-1}) \oplus \Big[(\mathbf{16},\mathbf{1})\Big](\alpha+\alpha^{-1})(\beta+\beta^{-1})\,.
\end{aligned}
\tag{6.3}
$$

The massless spectrum is found by tensoring the above left and right-moving states and keeping only invariant states. In this way, one finds a total content in the

[5] In the following, for simplicity we will not distinguish between $\mathbf{16}$ and $\overline{\mathbf{16}}$, $\mathbf{2}$ and $\overline{\mathbf{2}}$ representations. For the same reason, we will not report the $U(1)$ charges of the states. One can easily check that the spectrum is chiral.

untwisted sector which can be summarized as follows:

$$
\begin{aligned}
\mathbf{8_V} \otimes \mathbf{8_V} &: \mathbf{2_V} \otimes \mathbf{2_V} \,\oplus\, 10\,; \\
\mathbf{8_S} \otimes \mathbf{8_V} &: 4\,; \\
\mathbf{8_V} \otimes \mathbf{248} &: \mathbf{2_V} \otimes \Big[(\mathbf{45},\mathbf{1}) \oplus (\mathbf{1},\mathbf{3}) \oplus 2\,(\mathbf{1},\mathbf{1})\Big] \\
&\quad \oplus 4\,(\mathbf{10},\mathbf{2}) \,\oplus\, 2\,(\mathbf{10},\mathbf{1}) \,\oplus\, 4\,(\mathbf{1},\mathbf{1}) \,\oplus\, 2\,(\mathbf{16},\mathbf{2})\,; \\
\mathbf{8_S} \otimes \mathbf{248} &: 4\,(\mathbf{10},\mathbf{1}) \,\oplus\, 4\,(\mathbf{1},\mathbf{2}) \,\oplus\, 4\,(\mathbf{16},\mathbf{2}) \,\oplus\, 4\,(\mathbf{16},\mathbf{1})\,; \\
\mathbf{8_V} \otimes \mathbf{248}' &: \mathbf{2_V} \otimes \mathbf{248}'\,; \\
\mathbf{8_S} \otimes \mathbf{248}' &: -\,.
\end{aligned}
\tag{6.4}
$$

Consider next α_i-twisted sectors. The α_1-twisted sector preserves $N=2$ SUSY, and the spectrum of hypermultiplets is known: at each of the 9 fixed-planes one gets 1 **56** and 7 singlets of E_7. Each of the $\alpha_{2,3}$-twisted sectors preserves instead a $N=1$ SUSY, and the spectrum of chiral multiplets is similar for both of them: at each of the 27 fixed-points one gets a $(\mathbf{27},\mathbf{1})$ and 3 copies of $(\mathbf{1},\bar{\mathbf{3}})$ of $E_6 \times SU(3)$. Decomposing into representations of $SO(10)\times SU(2)$, one finds in total:

$$
\begin{aligned}
&\alpha_1 : 3 \text{ hyper-mult. in } \Big[2\,(\mathbf{16},\mathbf{1}) \,\oplus\, (\mathbf{10},\mathbf{2}) \,\oplus\, 2\,(\mathbf{1},\mathbf{2}) \,\oplus\, 7\,(\mathbf{1},\mathbf{1})\Big]\,; \\
&\alpha_2 : 9 \text{ chiral-mult. in } \Big[(\mathbf{16},\mathbf{1}) \,\oplus\, (\mathbf{10},\mathbf{1}) \,\oplus\, 3\,(\mathbf{1},\mathbf{2}) \,\oplus\, 4\,(\mathbf{1},\mathbf{1})\Big]\,; \\
&\alpha_3 : 9 \text{ chiral-mult. in } \Big[(\mathbf{16},\mathbf{1}) \,\oplus\, (\mathbf{10},\mathbf{1}) \,\oplus\, 3\,(\mathbf{1},\mathbf{2}) \,\oplus\, 4\,(\mathbf{1},\mathbf{1})\Big]\,.
\end{aligned}
$$

Finally, consider the β-twisted sectors, where potential tachyonic states might arise. In the left-moving sector, there is only one such state in the NS sector with $(p+v)^2=\frac{1}{9}$ and $N_L=0$. For right-movers, there are 14 such states with $(p'+v')^2=\frac{10}{9}$ and $N_R=0$. Pairing these states, one finds would-be tachyons in the $(\mathbf{10},\mathbf{1})\oplus 2\,(\mathbf{1},\mathbf{2})$ of $SO(10)\times SU(2)$ with $\frac{\alpha'}{2}m^2=-\frac{2}{3}+\frac{1}{2}(|P_L|^2[\hat{h}]+|P_R|^2[\hat{h}])$. The level-matching condition selects the KK and winding modes satisfying $m(n+h)=$ integer, allowing $m=0$ mod 3. The worst situation arises for $m=n=0$, and using $m_0^2=-\frac{4}{3\alpha'}$ and $U=e^{i\frac{\pi}{3}}$, one computes from (4.7) that $T_0=\sqrt{3}$. For $|T-i\,T_0|>T_0$, all the states in these sectors are massive[6].

Summarizing, the model we have constructed exhibits a chiral spectrum, and SUSY is broken at the scale $M_{\rm SUSY}=R_1^{-1}$ together with part of the gauge group. Tachyons can be avoided independently of R_1^{-1} by choosing $B_1>\sqrt{3}\,\alpha'$. One has $n_B-n_F=534$ from the untwisted sector. There are then 3 supersymmetric twisted

[6]Notice, however, that since these tachyons are in the appropriate representation to correspond to the SM Higgs, one may wish to keep some of them and impose a less restrictive constraint.

sectors, clearly with $n_B - n_F = 0$. Finally, there is one non-supersymmetric twisted sector, which gives $n_B - n_F = 0$ if $|T - i\,T_0| > T_0$ and $n_B - n_F = 14$ if $|T - i\,T_0| = T_0$.

6.2 More general embeddings

Models with more general embeddings can be easily constructed, and there are actually only a few possibilities to explore. As usual, thanks to the symmetries of the E_8 lattice, one can restrict shift vectors with length squared less than 1, whose embeddings must satisfy the conditions (6.1). Moreover, when applied 3 times, they must reduce to a lattice vector; since the lattice is even, this implies that $v'^2_\alpha, v''^2_\alpha, v'^2_\beta, v''^2_\beta = 0 \bmod \frac{4}{9}$. The first condition in (6.1) leaves then 9 independent possibilities for $(v'^2_\alpha, v''^2_\alpha)$, namely: $(0, \frac{2}{9})$, $(\frac{2}{9}, 0)$, $(0, \frac{8}{9})$, $(\frac{8}{9}, 0)$, $(\frac{2}{9}, \frac{2}{3})$, $(\frac{2}{3}, \frac{2}{9})$, $(\frac{4}{9}, \frac{4}{9})$, $(\frac{2}{3}, \frac{8}{9})$, $(\frac{8}{9}, \frac{2}{3})$. Similarly, the second condition in (6.1) restricts $(v'^2_\beta, v''^2_\beta)$ to be among the following 8 possibilities: $(\frac{2}{9}, \frac{2}{9})$, $(0, \frac{4}{9})$, $(\frac{4}{9}, 0)$, $(\frac{2}{3}, \frac{4}{9})$, $(\frac{4}{9}, \frac{2}{3})$, $(\frac{2}{9}, \frac{8}{9})$, $(\frac{8}{9}, \frac{2}{9})$, $(\frac{8}{9}, \frac{8}{9})$. As in [12], one can then choose a unique representative w for each value of w^2. Finally, the last condition in (6.1) turns out to constrain only the relative permutations of the shift vectors for the two factors.

We have computed the mismatch $n_B - n_F$ between massless bosons and fermions, as well as the number n_T of would-be tachyons, for all these models. The possible values for $(n_B - n_F, n_T)$ depend only on the embedding $(v'^2_\beta, v''^2_\beta)$ of the SUSY-breaking element β, and one finds $(318, 2)$ for $(\frac{2}{9}, \frac{2}{9})$, $(534, 14)$ for $(0, \frac{4}{9})$ or $(\frac{4}{9}, 0)$, $(48, 2)$ for $(\frac{2}{3}, \frac{4}{9})$ or $(\frac{4}{9}, \frac{2}{3})$, $(156, 8)$ for $(\frac{2}{9}, \frac{8}{9})$ or $(\frac{8}{9}, \frac{2}{9})$, and $(-6, 0)$ for $(\frac{8}{9}, \frac{8}{9})$. There exist therefore models without any possible tachyons[7]. Notice also that for generic embeddings, it becomes particularly clear that the class of models under consideration is intrinsically chiral. Indeed, each sector, and in particular the two $N = 1$ sectors, will have in general a different gauge twist, leading to distinct spectra of representations.

We did not consider the additional freedom of adding Wilson lines in our models. It should be appreciated, however, that like most of the other moduli, Wilson lines are now dynamical. A non-trivial effective potential is generated for these gauge-invariant operators [26], that will in general dynamically break part of the gauge group.

7. Conclusions

In this paper, a new class of four-dimensional non-supersymmetric string vacua has

[7] In these cases, one could define consistent models with hard SUSY breaking at the string scale by dropping the shift in the SUSY-breaking element β.

been analysed. The key point of the construction resides in the idea of considering freely acting translations and non-supersymmetric rotations, in addition to standard supersymmetric orbifold rotations. In this way, SUSY is broken at the compactification scale $M_{\rm c}$ through a string version of the Scherk–Schwarz mechanism [16]. Although we focused on simple examples of oriented closed string models, our construction is quite general and can be easily generalized in various way. For example, one can construct more complicated models with different supersymmetries or gauge symmetries being broken at different compactification scales $M_{\rm c}^i$. Unoriented models with D-branes and O-planes can be derived from Type IIB models as in [17, 18].

There are, we believe, several interesting issues that deserve further study. Among all, the most important would be a deeper analysis of the quantum stability of these models. In particular, one should study the quantum effective potential for the compactification moduli to see whether a stabilization of the geometry can be achieved. A similar question should be faced also for the dilaton and for Wilson lines, whose VEV's are also dynamically determined at the quantum level. Using the by now well established string web of dualities, it would also be instructive to analyse possible dual realizations of our models, allowing to study their strong coupling behaviour.

Finally, we think that the new SUSY-breaking geometries found in this paper are quite promising from the point of view of realistic model building. As already noticed in the introduction, it would be exciting to embed in this kind of string models the recently constructed higher-dimensional field theory models with SS SUSY and gauge symmetry breaking [19, 27]. The possibility of having an exponentially suppressed cosmological constant [8, 1] is also quite appealing in this context.

Acknowledgements

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A. θ-functions and modular invariance

Defining $q = \exp 2\pi i\tau$, one has

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1 - q^n)\,, \tag{A.1}$$

and

$$\theta\begin{bmatrix}a\\b\end{bmatrix}(\tau) = \sum_n q^{\frac{1}{2}(n+a)^2} e^{2\pi i(n+a)b}$$
$$= e^{2\pi iab} q^{\frac{a^2}{2}} \prod_{n=1}^{\infty}(1-q^n)(1+q^{n+a-\frac{1}{2}}e^{2\pi ib})(1+q^{n-a-\frac{1}{2}}e^{-2\pi ib}) , \qquad (A.2)$$

satisfying the periodicity property

$$\theta\begin{bmatrix}a+m\\b+n\end{bmatrix}(\tau) = e^{2\pi ina}\theta\begin{bmatrix}a\\b\end{bmatrix}(\tau) . \qquad (A.3)$$

Under modular transformations, these functions transform as follows:

$$\eta(\tau+1) = e^{i\frac{\pi}{12}}\,\eta(\tau) , \qquad (A.4)$$
$$\eta(-1/\tau) = \sqrt{-i\tau}\,\eta(\tau) , \qquad (A.5)$$

and

$$\theta\begin{bmatrix}a\\b\end{bmatrix}(\tau+1) = e^{-i\pi a(a-1)}\,\theta\begin{bmatrix}a\\a+b-\frac{1}{2}\end{bmatrix}(\tau) , \qquad (A.6)$$

$$\theta\begin{bmatrix}a\\b\end{bmatrix}(-1/\tau) = \sqrt{-i\tau}\,e^{2\pi iab}\,\theta\begin{bmatrix}b\\-a\end{bmatrix}(\tau) . \qquad (A.7)$$

Using the above formulae, the modular properties of the basic partition functions reported in section 3 are the following:

$$Z_B\left[\begin{array}{c|c}h&\hat{h}\\g&\hat{g}\end{array}\right](\tau+1) = Z_B\left[\begin{array}{c|c}h&\hat{h}\\g+h&\hat{g}+\hat{h}\end{array}\right](\tau) , \qquad (A.8)$$

$$Z_B\left[\begin{array}{c|c}h&\hat{h}\\g&\hat{g}\end{array}\right](-1/\tau) = Z_B\left[\begin{array}{c|c}g&\hat{g}\\Nv-h&1-\hat{h}\end{array}\right](\tau) , \qquad (A.9)$$

and

$$Z_F\left[\begin{array}{c|c}a&h\\b&g\end{array}\right](\tau+1) = e^{-i\frac{\pi}{12}-i\pi[a(a-1)+h^2]}Z_F\left[\begin{array}{c|c}a&h\\a+b-\frac{1}{2}&g+h\end{array}\right](\tau) , \qquad (A.10)$$

$$Z_F\left[\begin{array}{c|c}a&h\\b&g\end{array}\right](-1/\tau) = e^{-2\pi i[ab+g(Nv-h)+bNv]}Z_F\left[\begin{array}{c|c}b&g\\a&Nv-h\end{array}\right](\tau) . \qquad (A.11)$$

At this point, it is straightforward to derive the conditions required to achieve modular invariance of the general partition function (3.8). A basic modular transformation maps the $\left[{H \atop G}\right]$ sector into either the $\left[{H \atop G+H}\right]$ or $\left[{G \atop \bar{H}}\right]$ sectors, and since

$$N\begin{bmatrix}H\\G+H\end{bmatrix} = N\begin{bmatrix}G\\\bar{H}\end{bmatrix} = N\begin{bmatrix}H\\G\end{bmatrix} , \qquad (A.12)$$

the residual phases occurring in this transformation will severely constrain the $C\left[{H \atop G}\right]$'s.

For a generic Type IIB model, the partition function (3.9) in a generic sector is found to transform (using (3.3)) as:

$$Z\begin{bmatrix} H \\ G \end{bmatrix}(\tau+1) = Z\begin{bmatrix} H \\ G+H \end{bmatrix}(\tau)\,, \tag{A.13}$$

$$Z\begin{bmatrix} H \\ G \end{bmatrix}(-1/\tau) = Z\begin{bmatrix} G \\ \bar{H} \end{bmatrix}(\tau)\,. \tag{A.14}$$

One can therefore take $C\left[{H \atop G}\right] = 1$ as in (3.10), without any further condition.

For a generic heterotic $E_8 \times E_8$ model, again using (3.3), the partition function (3.11) transforms as:

$$Z\begin{bmatrix} H \\ G \end{bmatrix}(\tau+1) = e^{-i\pi(h^2-h'^2-h''^2)}\, Z\begin{bmatrix} H \\ G+H \end{bmatrix}(\tau)\,, \tag{A.15}$$

$$Z\begin{bmatrix} H \\ G \end{bmatrix}(-1/\tau) = e^{-2\pi i[g\cdot(Nv-h)-g'\cdot(Nv'-h')-g''\cdot(Nv''-h'')]}\, Z\begin{bmatrix} G \\ \bar{H} \end{bmatrix}(\tau)\,. \tag{A.16}$$

These transformations leave the partition function invariant if the $C\left[{H \atop G}\right]$'s satisfy

$$C\begin{bmatrix} H \\ G+H \end{bmatrix} = e^{-i\pi(h^2-h'^2-h''^2)}\, C\begin{bmatrix} H \\ G \end{bmatrix}, \tag{A.17}$$

$$C\begin{bmatrix} G \\ \bar{H} \end{bmatrix} = e^{-2\pi i[g\cdot(Nv-h)-g'\cdot(Nv'-h')-g''\cdot(Nv''-h'')]}\, C\begin{bmatrix} H \\ G \end{bmatrix}. \tag{A.18}$$

An additional consistency condition arises in this case from the requirement that each sector $\left[{H \atop G}\right]$ should be separately invariant under $\tau \to \tau + N$. This happens if $N(v^2 - v'^2 - v''^2) = 0 \bmod 2$, as anticipated in (3.13). In this case, all the phases proportional to N drop from (A.18), and the conditions (A.17) and (A.18) have the unique solution $C\left[{H \atop G}\right] = e^{-i\pi(g\cdot h - g'\cdot h' - g''\cdot h'')}$, reported in (3.12). This factor is identified with the total phase picked up by the vacuum $|\Omega[H]\rangle$ in the H-twisted sector under the orbifold action G. This can be verified explicitly by constructing this vacuum with twist fields. Notice that for standard embedding ($v' = v$, $v'' = 0$) this is equal to 1 in all sectors.

References

[1] I. Antoniadis, *Phys. Lett.* **B 246** (1990) 377.

[2] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B 429** (1998) 263, `hep-ph/9803315`;
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B 436** (1998) 257, `hep-ph/9804398`.

[3] I. Antoniadis, S. Dimopoulos and G. Dvali, *Nucl. Phys.* **B 516** (1998) 70, `hep-ph/9710204`.

[4] J. Scherk and J.H. Schwarz, *Phys. Lett.* **B 82** (1979) 60; *Nucl. Phys.* **B 153** (1979) 61.

[5] E. Cremmer, J. Scherk and J.H. Schwarz, *Phys. Lett.* **B 84** (1979) 83.

[6] P. Fayet, *Phys. Lett.* **B 159** (1985) 121; *Nucl. Phys.* **B 263** (1989) 75.

[7] T. Banks and L. Dixon, *Nucl. Phys.* **B 307** (1988) 93;
I. Antoniadis, C. Bachas, D. Lewellen and T. Tomaras, *Phys. Lett.* **B 207** (1988) 441.

[8] H. Itoyama and T.R. Taylor, *Phys. Lett.* **B 186** (1987) 129.

[9] V. Kaplunovsky, *Nucl. Phys.* **B 307** (1988) 145;
L.J. Dixon, V. Kaplunovsky and J. Louis, *Nucl. Phys.* **B 355** (1991) 649.

[10] E. Kiritsis, C. Kounnas, P.M. Petropoulos and J. Rizos, *Phys. Lett.* **B 385** (1996) 87, `hep-th/9606087`; *Nucl. Phys.* **B 540** (1999) 87, `hep-th/9807067`.

[11] I. Antoniadis, C. Muñoz and M. Quirós, *Nucl. Phys.* **B 397** (1993) 515;
I. Antoniadis and K. Benakli, *Phys. Lett.* **B 326** (1994) 69, `hep-ph/9310151`;
I. Antoniadis, K. Benakli and M. Quirós, *Phys. Lett.* **B 331** (1994) 313, `hep-ph/9403290`.

[12] L. Dixon, J.A. Harvey, C. Vafa and E. Witten, *Nucl. Phys.* **B 261** (1985) 678; *Nucl. Phys.* **B 274** (1986) 285.

[13] C. Kounnas and M. Porrati, *Nucl. Phys.* **B 310** (1988) 355;
S. Ferrara, C. Kounnas and M. Porrati, *Phys. Lett.* **B 206** (1988) 25;
S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner, *Nucl. Phys.* **B 318** (1989) 75;
C. Kounnas and B. Rostand, *Nucl. Phys.* **B 341** (1990) 641;
I. Antoniadis and C. Kounnas, *Phys. Lett.* **B 261** (1991) 369.

[14] K. Benakli, *Phys. Lett.* **B 386** (1996) 106, `hep-th/9509115`.

[15] E. Dudas and C. Grojean, *Nucl. Phys.* **B 507** (1997) 553, `hep-th/9704177`.

[16] E. Kiritsis and C. Kounnas, *Nucl. Phys.* **B 503** (1997) 117, `hep-th/9703059`.

[17] I. Antoniadis, E. Dudas and A. Sagnotti, *Nucl. Phys.* **B 544** (1999) 469, `hep-th/9807011`.

[18] I. Antoniadis, G. D'Appollonio, E. Dudas and A. Sagnotti, *Nucl. Phys.* **B 553** (1999) 133, `hep-th/9812118`; *Nucl. Phys.* **B 565** (2000) 123, `hep-th/9907184`;
A.L. Cotrone, *Mod. Phys. Lett.* **A14** (1999) 2487, `hep-th/9909116`.

[19] A. Pomarol and M. Quirós, *Phys. Lett.* **B 438** (1998) 255, `hep-ph/9806263`;
I. Antoniadis, S. Dimopoulos, A. Pomarol and M. Quiros, *Nucl. Phys.* **B 544** (1999) 503, `hep-ph/9810410`;
A. Delgado, A. Pomarol and M. Quirós, *Phys. Rev.* **D 60** (1999) 095008, `hep-ph/9812489`.

[20] R. Barbieri, L.J. Hall and Y. Nomura, *Phys. Rev.* **D 63** (2001) 105005, `hep-ph/0011311`.

[21] C. Vafa, *Nucl. Phys.* **B 273** (1986) 592.

[22] I. Senda and A. Sugamoto, *Phys. Lett.* **B 211** (1988) 308; *Nucl. Phys.* **B 302** (1988) 291.

[23] L. Ibáñez, J. Mas, H.P. Nilles and F. Quevedo, *Nucl. Phys.* **B 301** (1988) 157.

[24] A. Schellekens and N. Warner, *Nucl. Phys.* **B 287** (1987) 317;
W. Lerche, B. Nilsson, A. Schellekens and N. Warner, *Nucl. Phys.* **B 299** (1988) 91;
C.A. Scrucca and M. Serone, *J. High Energy Phys.* **0102** (2001) 019, `hep-th/0012124`.

[25] C.A. Scrucca and M. Serone, *Nucl. Phys.* **B 564** (2000) 555, `hep-th/9907112`; *J. High Energy Phys.* **9912** (1999) 024, `hep-th/9912108`; *J. High Energy Phys.* **0007** (2000) 025, `hep-th/0006201`.

[26] Y. Hosotani, *Phys. Lett.* **B 126** (1983) 309; *Phys. Lett.* **B 129** (1983) 193; *Ann. Phys. (NY)* **190** (1989) 233

[27] Y. Kawamura, *Prog. Theor. Phys.* **103** (2000) 613, `hep-ph/9902423`; "Triplet-doublet splitting, proton stability and extra dimension", `hep-ph/0012125`; *Prog. Theor. Phys.* **105** (2001) 691, `hep-ph/0012352`;
G. Altarelli and F. Feruglio, *Phys. Lett.* **B 511** (2001) 257, `hep-ph/0102301`;
L. Hall and Y. Nomura, "Gauge unification in higher dimensions," `hep-ph/0103125`;
A. Hebecker and J. March-Russel, "A minimal $S^1/(\mathbf{Z}_2 \times \mathbf{Z}'_2)$ orbifold GUT", `hep-ph/0106166`; "The structure of GUT breaking by orbifolding", `hep-ph/0107039`.